FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

Version 0
November 9, 2008

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Contents

Chapter 19

Electric Circuits - Grade 11

19.1 Introduction

The study of electrical circuits is essential to understand the technology that uses electricity in the real-world. This includes electricity being used for the operation of electronic devices like computers.

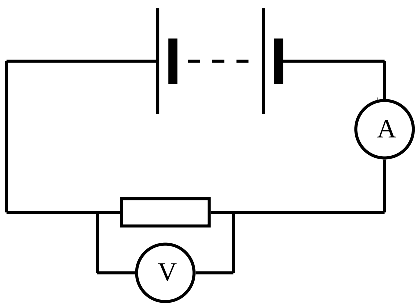
19.2 Ohm's Law

19.2.1 Definition of Ohm's Law

Activity :: Experiment : Ohm's Law

Aim:

In this experiment we will look at the relationship between the current going through a resistor and the potential difference (voltage) across the same resistor.



Method:

1. Set up the circuit according to the circuit diagram.
2. Draw the following table in your lab book.

Voltage, V (V)	Current, I (A)
1,5	
3,0	
4,5	
6,0	

3. Get your teacher to check the circuit before turning the power on.
4. Measure the current.
5. Add one more 1,5 V battery to the circuit and measure the current again.

6. Repeat until you have four batteries and you have completed your table.
7. Draw a graph of voltage versus current.

Results:

1. Does your experimental results verify Ohm's Law? Explain.
2. How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor R_0?

Activity :: Activity : Ohm's Law

If you do not have access to the equipment necessary for the Ohm's Law experiment, you can do this activity.

Voltage, V (V)	Current, I (A)
3,0	0,4
6,0	0,8
9,0	1,2
12,0	1,6

1. Plot a graph of voltage (on the y-axis) and current (on the x-axis).

Conclusions:

1. What type of graph do you obtain (straight line, parabola, other curve)
2. Calculate the gradient of the graph.
3. Does your experimental results verify Ohm's Law? Explain.
4. How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor R_0?

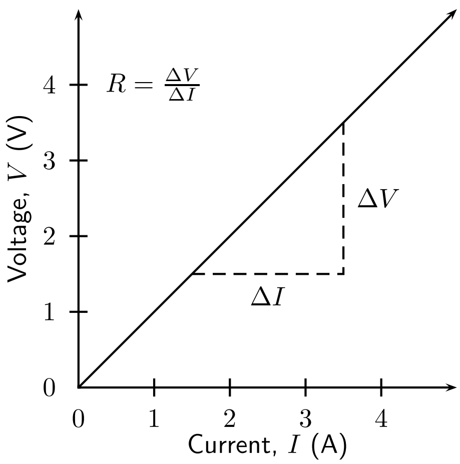
An important relationship between the current, voltage and resistance in a circuit was discovered by Georg Simon Ohm and is called **Ohm's Law**.

Definition: Ohm's Law

The amount of electric current through a metal conductor, at a constant temperature, in a circuit is proportional to the voltage across the conductor. Mathematically, Ohm's Law is written:

$$V = R \cdot I.$$

Ohm's Law tells us that if a conductor is at a constant temperature, the voltage across the ends of the conductor is proportional to the current. This means that if we plot voltage on the y-axis of a graph and current on the x-axis of the graph, we will get a straight-line. The gradient of the straight-line graph is then the resistance of the conductor.



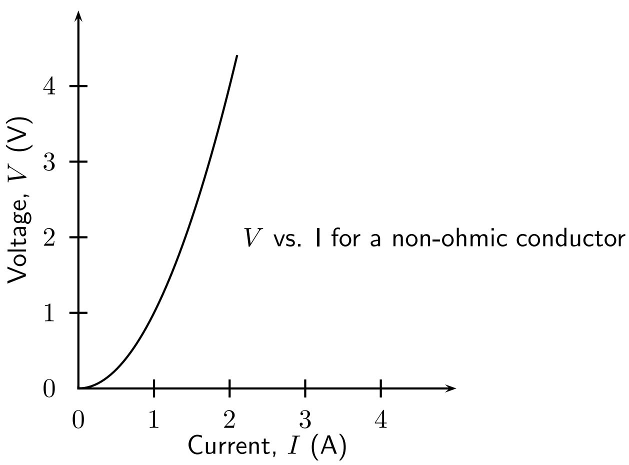
19.2.2 Ohmic and non-ohmic conductors

As you have seen, there is a mention of *constant temperature* when we talk about Ohm's Law. This is because the resistance of some conductors change as their temperature changes. These types of conductors are called *non-ohmic* conductors, because they do not obey Ohm's Law. As can be expected, the conductors that obey Ohm's Law are called *ohmic* conductors. A light bulb is a common example of a non-ohmic conductor. Nichrome wire is an ohmic conductor.

In a light bulb, the resistance of the filament wire will increase dramatically as it warms from room temperature to operating temperature. If we increase the supply voltage in a real lamp circuit, the resulting increase in current causes the filament to increase in temperature, which increases its resistance. This effectively limits the increase in current. In this case, voltage and current do not obey Ohm's Law.

The phenomenon of resistance changing with variations in temperature is one shared by almost all metals, of which most wires are made. For most applications, these changes in resistance are small enough to be ignored. In the application of metal lamp filaments, which increase a lot in temperature (up to about 1000°C, and starting from room temperature) the change is quite large.

In general non-ohmic conductors have plots of voltage against current that are curved, indicating that the resistance is not constant over all values of voltage and current.

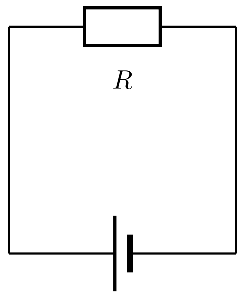


Activity :: Experiment : Ohmic and non-ohmic conductors
Repeat the experiment as decribed in the previous section. In this case use a light bulb as a resistor. Compare your results to the ohmic resistor.

19.2.3 Using Ohm's Law

We are now ready to see how Ohm's Law is used to analyse circuits.

Consider the circuit with an ohmic resistor, R. If the resistor has a resistance of 5 Ω and voltage across the resistor is 5V, then we can use Ohm's law to calculate the current flowing through the resistor.



Ohm's law is:

$$V = R \cdot I$$

which can be rearranged to:

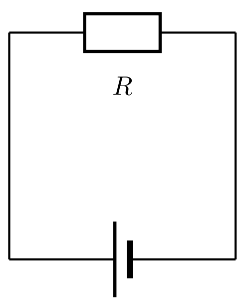
$$I = \frac{V}{R}$$

The current flowing in the resistor is:

$$\begin{aligned} I &= \frac{V}{R} \\ &= \frac{5\ \text{V}}{5\ \Omega} \\ &= 1\ \text{A} \end{aligned}$$

Worked Example 124: Ohm's Law

Question:



The resistance of the above resistor is 10 Ω and the current going through the resistor is 4 A. What is the potential difference (voltage) across the resistor?
Answer

Step 1 : Determine how to approach the problem
It is an Ohm's Law problem. So we use the equation:

$$V = R \cdot I$$

Step 2 : Solve the problem

$$\begin{aligned} V &= R \cdot I \\ &= (10)(4) \\ &= 40\ V \end{aligned}$$

Step 3 : Write the final answer
The voltage across the resistor is 40 V.

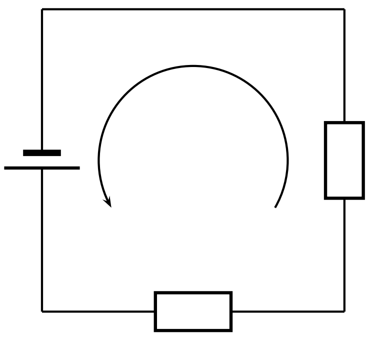
?

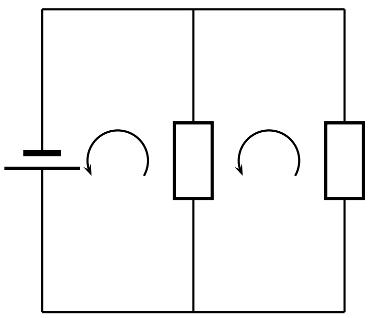
Exercise: Ohm's Law

1. Calculate the resistance of a resistor that has a potential difference of 8 V across it when a current of 2 A flows through it.
2. What current will flow through a resistor of 6 Ω when there is a potential difference of 18 V across its ends?
3. What is the voltage acroos a 10 Ω resistor when a current of 1,5 A flows though it?

19.3 Resistance

In Grade 10, you learnt about resistors and were introduced to circuits where resistors were connected in series and circuits where resistors were connected in parallel. In a series circuit there is one path for the current to flow through. In a parallel circuit there are multiple paths for the current to flow through.


series circuit
one current path

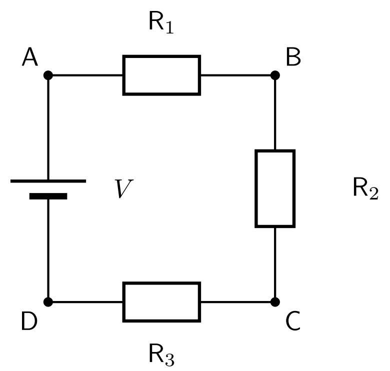

parallel circuit
multiple current paths

19.3.1 Equivalent resistance

When there is more than one resistor in a circuit, we are usually able to replace all resistors with a single resistor whose effect is the same as all the resistors put together. The resistance of the single resistor is known as *equivalent resistance*. We are able to calculate equivalent resistance for resistors connected in series and parallel.

Equivalent Series Resistance

Consider a circuit consisting of three resistors and a single battery connected in series.



The first principle to understand about series circuits is that the amount of current is the same through any component in the circuit. This is because there is only one path for electrons to flow in a series circuit. From the way that the battery is connected, we can tell which direction the current will flow. We know that charge flows from positive to negative, by convention. Current in this circuit will flow in a clockwise direction, from point A to B to C to D and back to A.

So, how do we use this knowledge to calculate the value of a single resistor that can replace the three resistors in the circuit and still have the same current?

We know that in a series circuit the current has to be the same in all components. So we can write:

$$I = I_1 = I_2 = I_3$$

We also know that total voltage of the circuit has to be equal to the sum of the voltages over all three resistors. So we can write:

$$V = V_1 + V_2 + V_3$$

Finally, we know that Ohm's Law has to apply for each resistor individually, which gives us:

$$\begin{aligned} V_1 &= I_1 \cdot R_1 \\ V_2 &= I_2 \cdot R_2 \\ V_3 &= I_3 \cdot R_3 \end{aligned}$$

Therefore:

$$V = I_1 \cdot R_1 + I_2 \cdot R_2 + I_3 \cdot R_3$$

However, because

$$I = I_1 = I_2 = I_3$$

, we can further simplify this to:

$$\begin{aligned} V &= I \cdot R_1 + I \cdot R_2 + I \cdot R_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

Further, we can write an Ohm's Law relation for the entire circuit:

$$V = I \cdot R$$

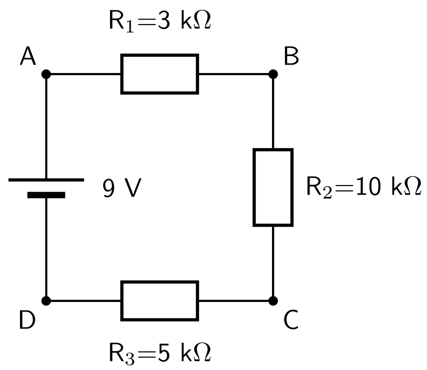
Therefore:

$$\begin{aligned} V &= I(R_1 + R_2 + R_3) \\ I \cdot R &= I(R_1 + R_2 + R_3) \\ \therefore \quad R &= R_1 + R_2 + R_3 \end{aligned}$$

Definition: Equivalent resistance in a series circuit, R_s
For n resistors in series the equivalent resistance is:

$$R_s = R_1 + R_2 + R_3 + \cdots + R_n$$

Let us apply this to the following circuit.



The resistors are in series, therefore:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 3\,\Omega + 10\,\Omega + 5\,\Omega \\ &= 18\,\Omega \end{aligned}$$

Worked Example 125: Equivalent series resistance I

Question: Two 10 kΩ resistors are connected in series. Calculate the equivalent resistance.

Answer

Step 1 : Determine how to approach the problem

Since the resistors are in series we can use:

$$R_s = R_1 + R_2$$

Step 2 : Solve the problem

$$\begin{aligned} R_s &= R_1 + R_2 \\ &= 10\,\text{k}\,\Omega + 10\,\text{k}\,\Omega \\ &= 20\,\text{k}\,\Omega \end{aligned}$$

Step 3 : Write the final answer

The equivalent resistance of two 10 kΩ resistors connected in series is 20 kΩ.

Worked Example 126: Equivalent series resistance II

Question: Two resistors are connected in series. The equivalent resistance is 100 Ω. If one resistor is 10 Ω, calculate the value of the second resistor.

Answer

Step 1 : Determine how to approach the problem

Since the resistors are in series we can use:

$$R_s = R_1 + R_2$$

We are given the value of R_s and R_1.

Step 2 : Solve the problem

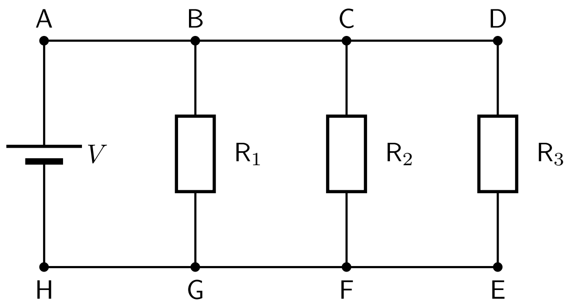
$$\begin{aligned} R_s &= R_1 + R_2 \\ \therefore \quad R_2 &= R_s - R_1 \\ &= 100\,\Omega - 10\,\Omega \\ &= 90\,\Omega \end{aligned}$$

Step 3 : Write the final answer

The second resistor has a resistance of 90Ω.

Equivalent parallel resistance

Consider a circuit consisting of a single battery and three resistors that are connected in parallel.



The first principle to understand about parallel circuits is that the voltage is equal across all components in the circuit. This is because there are only two sets of electrically common points in a parallel circuit, and voltage measured between sets of common points must always be the same at any given time. So, for the circuit shown, the following is true:

$$V = V_1 = V_2 = V_3$$

The second principle for a parallel circuit is that all the currents through each resistor must add up to the total current in the circuit.

$$I = I_1 + I_2 + I_3$$

Also, from applying Ohm's Law to the entire circuit, we can write:

$$V = \frac{I}{R_p}$$

where R_p is the equivalent resistance in this parallel arrangement.

We are now ready to apply Ohm's Law to each resistor, to get:

$$\begin{aligned} V_1 &= R_1 \cdot I_1 \\ V_2 &= R_2 \cdot I_2 \\ V_3 &= R_3 \cdot I_3 \end{aligned}$$

This can be also written as:

$$\begin{aligned} I_1 &= \frac{V_1}{R_1} \\ I_2 &= \frac{V_2}{R_2} \\ I_3 &= \frac{V_3}{R_3} \end{aligned}$$

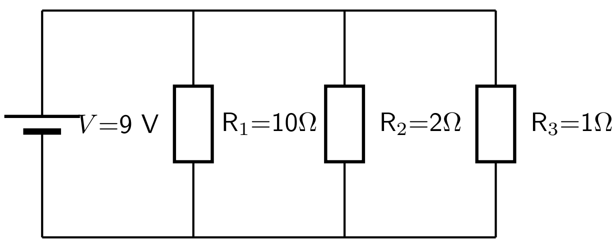
Now we have:

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ \frac{V}{R_p} &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \\ &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ \text{because} & \quad V = V_1 = V_2 = V_3 \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\ \therefore \quad \frac{1}{R_p} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \end{aligned}$$

Definition: Equivalent resistance in a parallel circuit, R_p
For n resistors in parallel, the equivalent resistance is:

$$\frac{1}{R_p} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right)$$

Let us apply this formula to the following circuit.



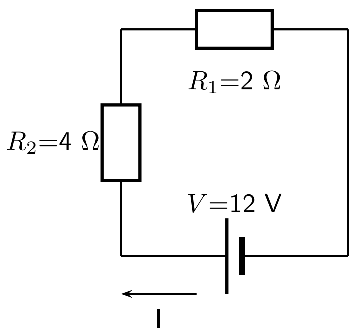
$$\begin{aligned} \frac{1}{R_p} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\ &= \left(\frac{1}{10\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{1\,\Omega}\right) \\ &= \left(\frac{1+5+10}{10}\right) \\ &= \left(\frac{16}{10}\right) \\ \therefore \quad R_p &= \frac{10}{16}\,\Omega \end{aligned}$$

19.3.2 Use of Ohm's Law in series and parallel Circuits

Worked Example 127: Ohm's Law

Question: Calculate the current (I) in this circuit if the resistors are both ohmic in nature.

Answer



Step 1 : Determine what is required

We are required to calculate the current flowing in the circuit.

Step 2 : Determine how to approach the problem

Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.

Step 3 : Find total resistance in circuit

Since the resistors are connected in series, the total resistance R is:

$$R = R_1 + R_2$$

Therefore,

$$R = 2 + 4 = 6\ \Omega$$

Step 4 : Apply Ohm's Law

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad I &= \frac{V}{R} \\ &= \frac{12}{6} \\ &= 2\ \mathrm{A} \end{aligned}$$

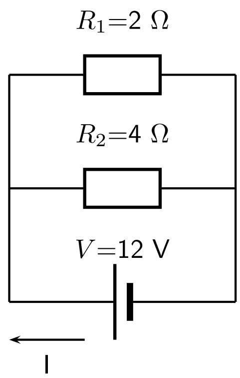
Step 5 : Write the final answer

A 2 A current is flowing in the circuit.

Worked Example 128: Ohm's Law I

Question: Calculate the current (I) in this circuit if the resistors are both ohmic in nature.

Answer



Step 1 : Determine what is required
We are required to calculate the current flowing in the circuit.
Step 2 : Determine how to approach the problem
Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.
Step 3 : Find total resistance in circuit
Since the resistors are connected in parallel, the total resistance R is:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Therefore,

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{2} + \frac{1}{4} \\ &= \frac{2+1}{4} \\ &= \frac{3}{4} \\ Therefore, R &= \frac{4}{3}\ \Omega \end{aligned}$$

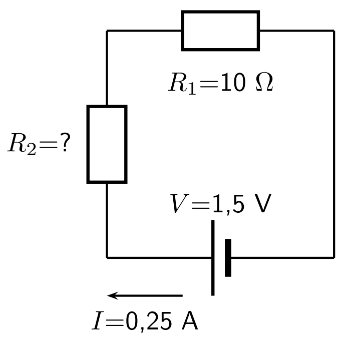
Step 4 : Apply Ohm's Law

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad I &= \frac{V}{R} \\ &= \frac{12}{\frac{4}{3}} \\ &= 9\ \mathrm{A} \end{aligned}$$

Step 5 : Write the final answer
A 9 A current is flowing in the circuit.

Worked Example 129: Ohm's Law II

Question: Two ohmic resistors (R_1 and R_2) are connected in series with a battery. Find the resistance of R_2, given that the current flowing through R_1 and R_2 is 0,25 A and that the voltage across the battery is 1,5 V. R_1=1 Ω.
Answer
Step 6 : Draw the circuit and fill in all known values.



Step 7 : Determine how to approach the problem.

We can use Ohm's Law to find the total resistance R in the circuit, and then calculate the unknown resistance using:

$$R = R_1 + R_2$$

in a series circuit.

Step 8 : Find the total resistance

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad R &= \frac{V}{I} \\ &= \frac{1{,}5}{0{,}25} \\ &= 6\,\Omega \end{aligned}$$

Step 9 : Find the unknown resistance

We know that:

$$R = 6\,\Omega$$

and that

$$R_1 = 1\,\Omega$$

Since

$$R = R_1 + R_2$$

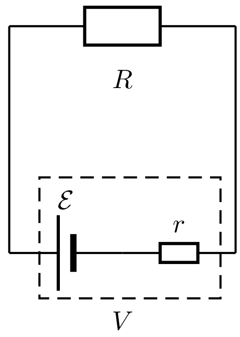
$$R_2 = R - R_1$$

Therefore,

$$R_2 = 5\,\Omega$$

19.3.3 Batteries and internal resistance

Real batteries are made from materials which have resistance. This means that real batteries are not just sources of potential difference (voltage), but they also possess internal resistances. If the pure voltage source is referred to as the emf, $\mathcal{E}$, then a real battery can be represented as an emf connected in series with a resistor r. The internal resistance of the battery is represented by the symbol r.



Definition: Load
The external resistance in the circuit is referred to as the load.

Suppose that the battery (or cell) with emf $\mathcal{E}$ and internal resistance r supplies a current I through an external load resistor R. Then the voltage drop across the load resistor is that supplied by the battery:

$$V = I \cdot R$$

Similarly, from Ohm's Law, the voltage drop across the internal resistance is:

$$V_r = I \cdot r$$

The voltage V of the battery is related to its emf $\mathcal{E}$ and internal resistance r by:

$$\begin{aligned} \mathcal{E} &= V + Ir; or \\ V &= \mathcal{E} - Ir \end{aligned}$$

The emf of a battery is essentially constant because it only depends on the chemical reaction (that converts chemical energy into electrical energy) going on inside the battery. Therefore, we can see that the voltage across the terminals of the battery is dependent on the current drawn by the load. The higher the current, the lower the voltage across the terminals, because the emf is constant. By the same reasoning, the voltage only equals the emf when the current is very small.

The maximum current that can be drawn from a battery is limited by a critical value I_c. At a current of I_c, V=0 V. Then, the equation becomes:

$$\begin{aligned} 0 &= \mathcal{E} - I_c r \\ I_c r &= \mathcal{E} \\ I_c &= \frac{\mathcal{E}}{r} \end{aligned}$$

The maximum current that can be drawn from a battery is less than $\frac{\mathcal{E}}{r}$.

Worked Example 130: Internal resistance

Question: What is the internal resistance of a battery if its emf is 12 V and the voltage drop across its terminals is 10 V when a current of 4 A flows in the circuit when it is connected across a load?

Answer

Step 1 : Determine how to approach the problem

It is an internal resistance problem. So we use the equation:

$$\mathcal{E} = V + Ir$$

Step 2 : Solve the problem

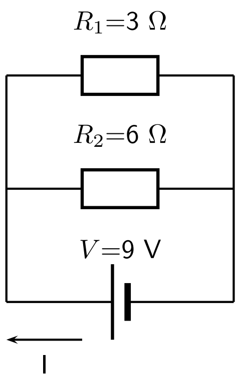
$$\begin{aligned}\mathcal{E} &= V + Ir \\ 12 &= 10 + 4(r) \\ &= 0.5\end{aligned}$$

Step 3 : Write the final answer
The internal resistance of the resistor is 0.5 Ω.

?

Exercise: Resistance

1. Calculate the equivalent resistance of:
   A three 2 Ω resistors in series;
   B two 4 Ωresistors in parallel;
   C a 4 Ω resistor in series with a 8 Ω resistor;
   D a 6 Ω resistor in series with two resistors (4 Ω and 2Ω) in parallel.
2. Calculate the current in this circuit if both resistors are ohmic.



3. Two ohmic resistors are connected in series. The resistance of the one resistor is 4 Ω̇What is the resistance of the other resistor if a current of 0,5 A flows through the resistors when they are connected to a voltage supply of 6 V.
4. Describe what is meant by the *internal resistance* of a real battery.
5. Explain why there is a difference between the emf and terminal voltage of a battery if the load (external resistance in the circuit) is comparable in size to the battery's internal resistance
6. What is the internal resistance of a battery if its emf is 6 V and the voltage drop across its terminals is 5,8 V when a current of 0,5 A flows in the circuit when it is connected across a load?

19.4 Series and parallel networks of resistors

Now that you know how to handle simple series and parallel circuits, you are ready to tackle problems like this:

It is relatively easy to work out these kind of circuits because you use everything you have already learnt about series and parallel circuits. The only difference is that you do it in stages.

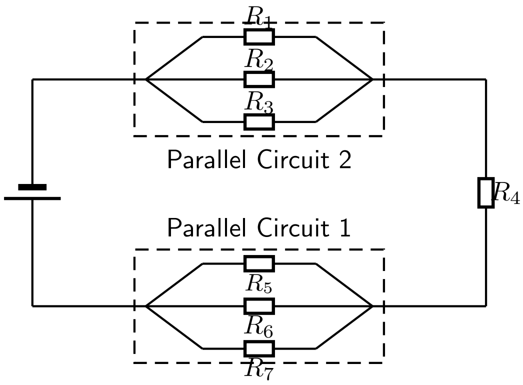
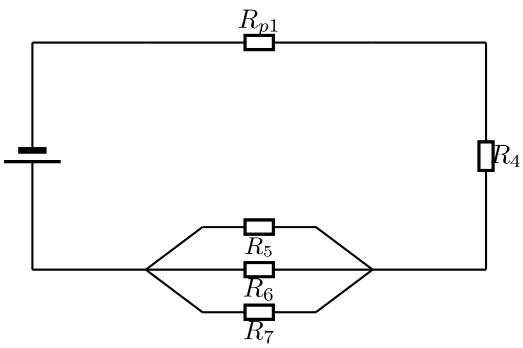


Figure 19.1: An example of a series-parallel network. The dashed boxes indicate parallel sections of the circuit.

In Figure 19.1, the circuit consists of 2 parallel portions that are then in series with 1 resistor. So, in order to work out the equivalent resistance, you start by reducing the parallel portions to a single resistor and then add up all the resistances in series. If all the resistors in Figure 19.1 had resistances of 10 Ω, we can calculate the equivalent resistance of the entire circuit.

We start by reducing *Parallel Circuit 1* to a single resistor.

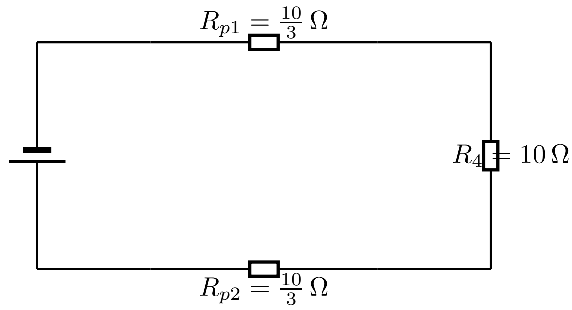


The value of R_{p1} is:

$$\begin{aligned}
\frac{1}{R_{p1}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
R_{p1} &= \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right)^{-1} \\
&= \left(\frac{1+1+1}{10}\right)^{-1} \\
&= \left(\frac{3}{10}\right)^{-1} \\
&= \frac{10}{3}\,\Omega
\end{aligned}$$

We can similarly replace *Parallel Circuit 2* with R_{p2} which has a value given by:

$$\begin{aligned}
\frac{1}{R_{p2}} &= \frac{1}{R_5}+\frac{1}{R_6}+\frac{1}{R_7} \\
R_{p2} &= \left(\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)^{-1} \\
&= \left(\frac{1+1+1}{10}\right)^{-1} \\
&= \left(\frac{3}{10}\right)^{-1} \\
&= \frac{10}{3}\,\Omega
\end{aligned}$$



This is now a simple series circuit and the equivalent resistance is:

$$\begin{aligned}
R &= R_{p1}+R_4+R_{p2} \\
&= \frac{10}{3}+10+\frac{10}{3} \\
&= \frac{100+30+100}{30} \\
&= \frac{230}{30} \\
&= 7\frac{2}{3}\,\Omega
\end{aligned}$$

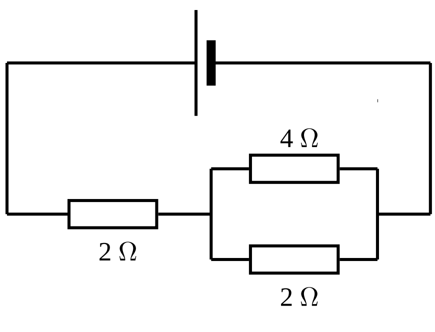
The equivalent resistance of the circuit in Figure 19.1 is $7\frac{2}{3}\,\Omega$.

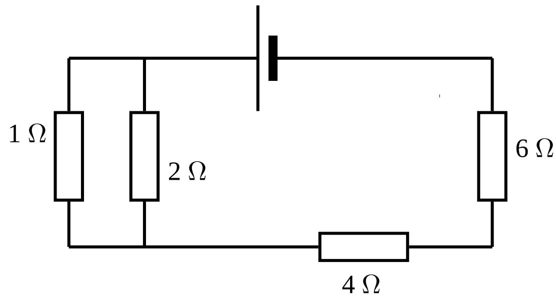
?

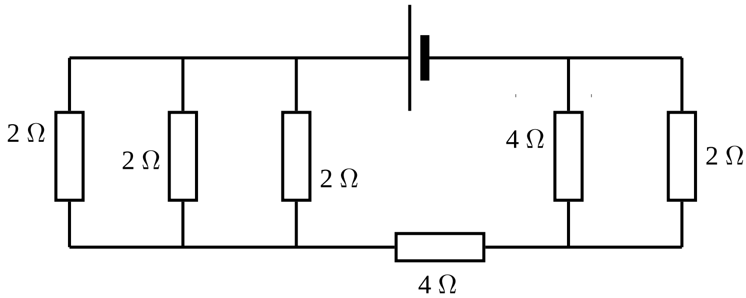
Exercise: Series and parallel networks

Determine the equivalent resistance of the following circuits:

1. Hello



2. 

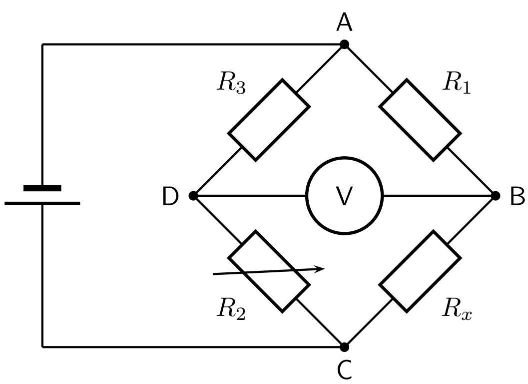
3. 

19.5 Wheatstone bridge

Another method of finding an unknown resistance is to use a *Wheatstone bridge*. A Wheatstone bridge is a measuring instrument that is used to measure an unknown electrical resistance by balancing two legs of a bridge circuit, one leg of which includes the unknown component. Its operation is similar to the original potentiometer except that in potentiometer circuits the meter used is a sensitive galvanometer.

Interesting Fact

The Wheatstone bridge was invented by Samuel Hunter Christie in 1833 and improved and popularized by Sir Charles Wheatstone in 1843.

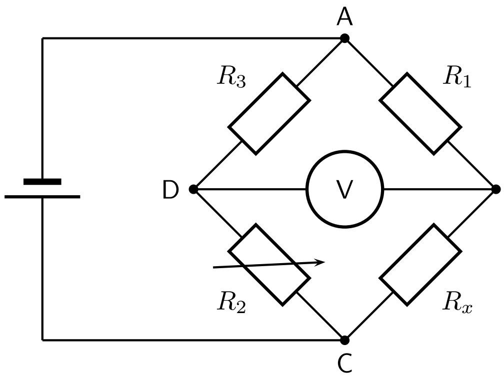


Circuit for Wheatstone bridge

In the circuit of the Wheatstone bridge, R_x is the unknown resistance. R_1, R_2 and R_3 are resistors of known resistance and the resistance of R_2 is adjustable. If the ratio of R_2:R_1 is equal to the ratio of R_x:R_3, then the voltage between the two midpoints will be zero and no current will flow between the midpoints. In order to determine the unknown resistance, R_2 is varied until this condition is reached. That is when the voltmeter reads 0 V.

Worked Example 131: Wheatstone bridge

Question:
Answer
What is the resistance of the unknown resistor R_x in the diagram below if R_1=4Ω R_2=8Ω and R_3=6Ω.



Circuit for Wheatstone bridge

Step 1 : Determine how to approach the problem
The arrangement is a Wheatstone bridge. So we use the equation:

$$R_x : R_3 = R_2 : R_1$$

Step 2 : Solve the problem

$$\begin{aligned} R_x : R_3 &= R_2 : R_1 \\ R_x : 6 &= 8 : 4 \\ R_x &= 12\ \Omega \end{aligned}$$

Step 3 : Write the final answer
The resistance of the unknown resistor is 12 Ω.

Extension: Power in electric circuits
In addition to voltage and current, there is another measure of free electron activity in a circuit: *power*. Power is a measure of how rapidly a standard amount of *work* is done. In electric circuits, power is a function of both voltage and current:

Definition: Electrical Power
Electrical power is calculated as:
$$P = I \cdot V$$

Power (P) is exactly equal to current (I) multiplied by voltage (V) and there is no extra constant of proportionality. The unit of measurement for power is the *Watt* (abbreviated W).

Interesting Fact

It was James Prescott Joule, not Georg Simon Ohm, who first discovered the mathematical relationship between power dissipation and current through a resistance. This discovery, published in 1841, followed the form of the equation:

$$P = I^2R$$

and is properly known as Joule's Law. However, these power equations are so commonly associated with the Ohm's Law equations relating voltage, current, and resistance that they are frequently credited to Ohm.

Activity :: Investigation : Equivalence
Use Ohm's Law to show that:

$$P = VI$$

is identical to

$$P = I^2R$$

and

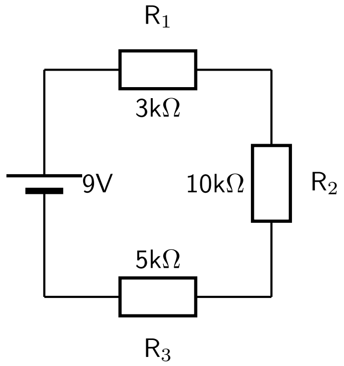
$$P = \frac{V^2}{R}$$

19.6 Summary

1. Ohm's Law states that the amount of current through a conductor, at constant temperature, is proportional to the voltage across the resistor. Mathematically we write $V = R/cdotI$
2. Conductors that obey Ohm's Law are called ohmic conductors; those who do not are called non-ohmic conductors.
3. We use Ohm's Law to calculate the resistance of a resistor. $(R = \frac{V}{I}$
4. The equivalent resistance of resistors in series (R_s) can be calculated as follows: $R_s = R_1 + R_2 + R_3 + ... + R_n$
5. The equivalent resistance of resistors in parallel (R_p) can be calculated as follows: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$
6. Real batteries have an internal resistance.
7. Wheatstone bridges can be used to accurately determine the resistance of an unknown resistor.

19.7 End of chapter exercise

1. Calculate the current in the following circuit and then use the current to calculate the voltage drops across each resistor.



2. Explain why a voltmeter is placed in parallel with a resistor.

3. Explain why an ammeter is placed in series with a resistor.

4. [IEB 2001/11 HG1] - **Emf**

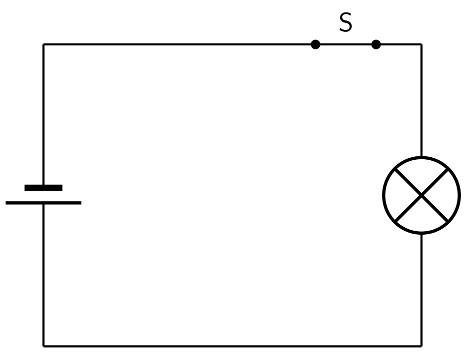
   A Explain the meaning of each of these two statements:

      i. "The current through the battery is 50 mA."

      ii. "The emf of the battery is 6 V."

   B A battery tester measures the current supplied when the battery is connected to a resistor of 100 Ω. If the current is less than 50 mA, the battery is "flat" (it needs to be replaced). Calculate the maximum internal resistance of a 6 V battery that will pass the test.

5. [IEB 2005/11 HG] The electric circuit of a torch consists of a cell, a switch and a small light bulb.



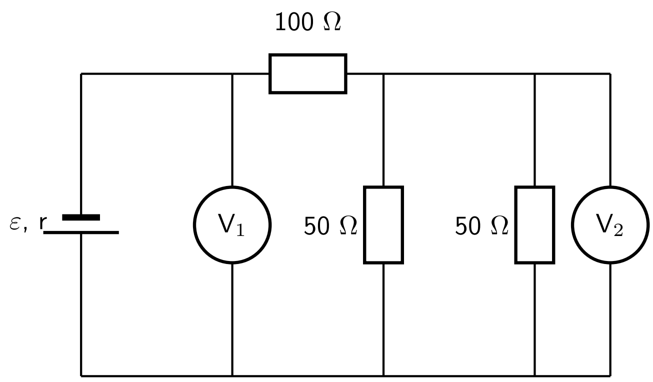
The electric torch is designed to use a D-type cell, but the only cell that is available for use is an AA-type cell. The specifications of these two types of cells are shown in the table below:

Cell	**emf**	**Appliance for which it is designed**	**Current drawn from cell when connected to the appliance for which it is designed**
D	1,5 V	torch	300 mA
AA	1,5 V	TV remote control	30 mA

What is likely to happen and why does it happen when the AA-type cell replaces the D-type cell in the electric torch circuit?

	What happens	Why it happens
(a)	the bulb is dimmer	the AA-type cell has greater internal resistance
(b)	the bulb is dimmer	the AA-type cell has less internal resistance
(c)	the brightness of the bulb is the same	the AA-type cell has the same internal resistance
(d)	the bulb is brighter	the AA-type cell has less internal resistance

6. [IEB 2005/11 HG1] A battery of emf ε and internal resistance r = 25 Ω is connected to this arrangement of resistors.



The resistances of voltmeters V_1 and V_2 are so high that they do not affect the current in the circuit.

A Explain what is meant by "the emf of a battery".
The power dissipated in the 100 Ω resistor is 0,81 W.

B Calculate the current in the 100 Ω resistor.

C Calculate the reading on voltmeter V_2.

D Calculate the reading on voltmeter V_1.

E Calculate the emf of the battery.

7. [SC 2003/11] A kettle is marked 240 V; 1 500 W.

A Calculate the resistance of the kettle when operating according to the above specifications.

B If the kettle takes 3 minutes to boil some water, calculate the amount of electrical energy transferred to the kettle.

8. [IEB 2001/11 HG1] - **Electric Eels**

Electric eels have a series of cells from head to tail. When the cells are activated by a nerve impulse, a potential difference is created from head to tail. A healthy electric eel can produce a potential difference of 600 V.

A What is meant by "a potential difference of 600 V"?

B How much energy is transferred when an electron is moved through a potential difference of 600 V?

Appendix A

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