



FHSST Authors

**The Free High School Science Texts:
Textbooks for High School Students
Studying the Sciences
Physics
Grades 10 - 12**

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Chapter 16

The Physics of Music - Grade 11

16.1 Introduction

What is your favorite musical instrument? How do you play it? Do you pluck a string, like a guitar? Do you blow through it, like a flute? Do you hit it, like a drum? All of these work by making standing waves. Each instrument has a unique sound because of the special waves made in it. These waves could be in the strings of a guitar or violin. They could also be in the skin of a drum or a tube of air in a trumpet. These waves are picked up by the air and later reach your ear as sound.

In Grade 10, you learned about standing waves and boundary conditions. We saw a rope that was:

- fixed at both ends
- fixed at one end and free at the other

We also saw a pipe:

- closed at both ends
- open at both ends
- open at one end, closed at the other

String and wind instruments are good examples of standing waves on strings and pipes.

One way to describe standing waves is to count nodes. Recall that a node is a point on a string that does not move as the wave changes. The anti-nodes are the highest and lowest points on the wave. There is a node at each end of a fixed string. There is also a node at the closed end of a pipe. But an open end of a pipe has an anti-node.

What causes a standing wave? There are incident and reflected waves traveling back and forth on our string or pipe. For some frequencies, these waves combine in just the right way so that the whole wave appears to be standing still. These special cases are called harmonic frequencies, or **harmonics**. They depend on the length and material of the medium.



Definition: Harmonic

A **harmonic** frequency is a frequency at which standing waves can be made.

16.2 Standing Waves in String Instruments

Let us look at a basic "instrument": a string pulled tight and fixed at both ends. When you pluck the string, you hear a certain pitch. This pitch is made by a certain frequency. What causes the string to emit sounds at this pitch?

You have learned that the frequency of a standing wave depends on the length of the wave. The wavelength depends on the nodes and anti-nodes. The longest wave that can "fit" on the string is shown in Figure 16.1. This is called the **fundamental** or **natural frequency** of the string. The string has nodes at both ends. The wavelength of the fundamental is twice the length of the string.

Now put your finger on the center of the string. Hold it down gently and pluck it. The standing wave now has a node in the middle of the string. There are three nodes. We can fit a whole wave between the ends of the string. This means the wavelength is equal to the length of the string. This wave is called the first harmonic. As we add more nodes, we find the second harmonic, third harmonic, and so on. We must keep the nodes equally spaced or we will lose our standing wave.

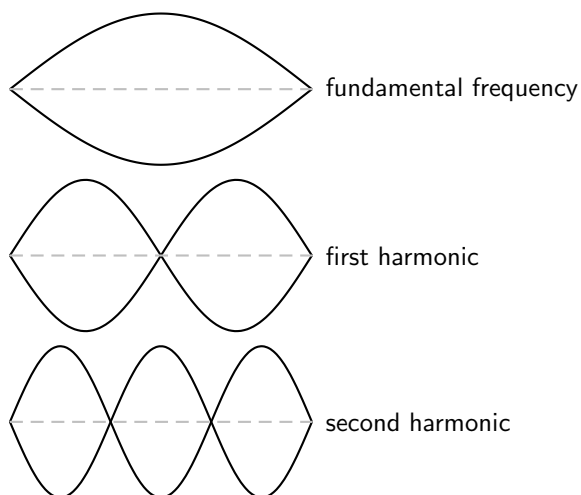


Figure 16.1: Harmonics on a string fixed at both ends.

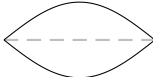

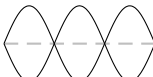
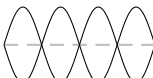
Activity :: Investigation : Waves on a String Fixed at Both Ends

This chart shows various waves on a string. The string length L is the dashed line.

1. Fill in the:

- number of nodes
- number of anti-nodes
- wavelength in terms of L

The first and last waves are done for you.

Wave	Nodes	Antinodes	Wavelength
	2	1	$2L$
			
			
	5	4	$\frac{L}{2}$

2. Use the chart to find a formula for the wavelength in terms of the number of nodes.

You should have found this formula:

$$\lambda = \frac{2L}{n-1}$$

Here, n is the number of nodes. L is the length of the string. The frequency f is:

$$f = \frac{v}{\lambda}$$

Here, v is the velocity of the wave. This may seem confusing. The wave is a *standing* wave, so how can it have a velocity? But one standing wave is made up of many waves that travel back and forth on the string. Each of these waves has the same velocity. This speed depends on the mass and tension of the string.



Worked Example 105: Harmonics on a String

Question: We have a standing wave on a string that is 65 cm long. The wave has a velocity of $143 \text{ m}\cdot\text{s}^{-1}$. Find the frequencies of the fundamental, first, second, and third harmonics.

Answer

Step 1 : Identify what is given and what is asked:

$$L = 65 \text{ cm} = 0.65 \text{ m}$$

$$v = 143 \text{ m}\cdot\text{s}^{-1}$$

$$f = ?$$

To find the frequency we will use $f = \frac{v}{\lambda}$

Step 2 : Find the wavelength for each harmonic:

To find f we need the wavelength of each harmonic ($\lambda = \frac{2L}{n-1}$). The wavelength is then substituted into $f = \frac{v}{\lambda}$ to find the harmonics. Table ?? below shows the calculations.

	Nodes	Wavelength $\lambda = \frac{2L}{n-1}$	Frequency $f = \frac{v}{\lambda}$
Fundamental frequency f_0	2	$\frac{2(0.65)}{2-1} = 1,3$	$\frac{143}{1,3} = 110 \text{ Hz}$
First harmonic f_1	3	$\frac{2(0.65)}{3-1} =$	$\frac{143}{2} = 220 \text{ Hz}$
Second harmonic f_2	4	$\frac{2(0.65)}{4-1} =$	$\frac{143}{3} = 330 \text{ Hz}$
Third harmonic f_3	5	$\frac{2(0.65)}{5-1} =$	$\frac{143}{4} = 440 \text{ Hz}$

110 Hz is the natural frequency of the A string on a guitar. The third harmonic, at 440 Hz, is the note that orchestras use for tuning.



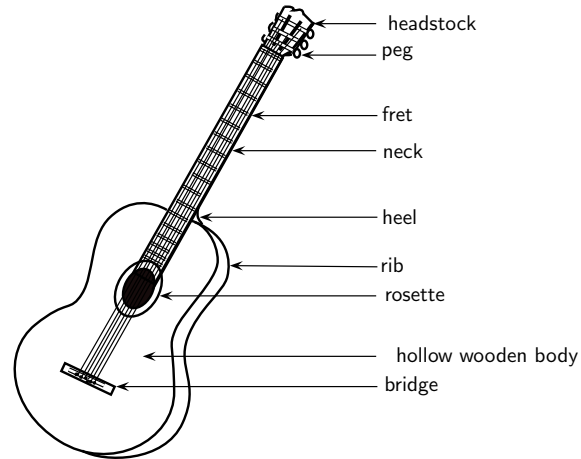
Extension: Guitar

Guitars use strings with high tension. The length, tension and mass of the strings affect the pitches you hear. High tension and short strings make high frequencies; Low tension and long strings make low frequencies. When a string is first plucked, it vibrates at many frequencies. All of these except the harmonics are quickly filtered out. The harmonics make up the tone we hear.

The body of a guitar acts as a large wooden soundboard. Here is how a soundboard works: the body picks up the vibrations of the strings. It then passes these vibrations to the air. A sound hole allows the soundboard of the guitar to vibrate more freely. It also helps sound waves to get out of the body.

The neck of the guitar has thin metal bumps on it called frets. Pressing a string against a fret shortens the length of that string. This raises the natural frequency and the pitch of that string.

Most guitars use an "equal tempered" tuning of 12 notes per octave. A 6 string guitar has a range of $4 \frac{1}{2}$ octaves with pitches from 82.407 Hz (low E) to 2093 kHz (high C). Harmonics may reach over 20 kHz, in the inaudible range.

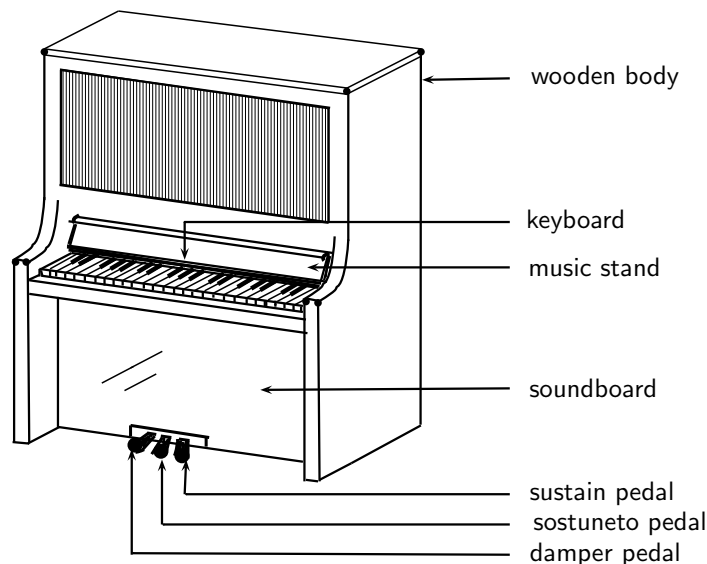


Extension: Piano

Let us look at another stringed instrument: the piano. The piano has strings that you can not see. When a key is pressed, a felt-tipped hammer hits a string inside the piano. The pitch depends on the length, tension and mass of the string. But there are many more strings than keys on a piano. This is because the short and thin strings are not as loud as the long and heavy strings. To make up for this, the higher keys have groups of two to four strings each.

The soundboard in a piano is a large cast iron plate. It picks up vibrations from the strings. This heavy plate can withstand over 200 tons of pressure from string tension! Its mass also allows the piano to sustain notes for long periods of time.

The piano has a wide frequency range, from 27,5 Hz (low A) to 4186,0 Hz (upper C). But these are just the fundamental frequencies. A piano plays complex, rich tones with over 20 harmonics per note. Some of these are out of the range of human hearing. Very low piano notes can be heard mostly because of their higher harmonics.



16.3 Standing Waves in Wind Instruments

A wind instrument is an instrument that is usually made with a pipe or thin tube. Examples of wind instruments are recorders, clarinets, flutes, organs etc.

When one plays a wind instrument, the air that is pushed through the pipe vibrates and standing waves are formed. Just like with strings, the wavelengths of the standing waves will depend on the length of the pipe and whether it is open or closed at each end. Let's consider each of the following situations:

- A pipe with both ends open, like a flute or organ pipe.
- A pipe with one end open and one closed, like a clarinet.

If you blow across a small hole in a pipe or reed, it makes a sound. If both ends are open, standing waves will form according to figure 16.2. You will notice that there is an anti-node at each end. In the next activity you will find how this affects the wavelengths.

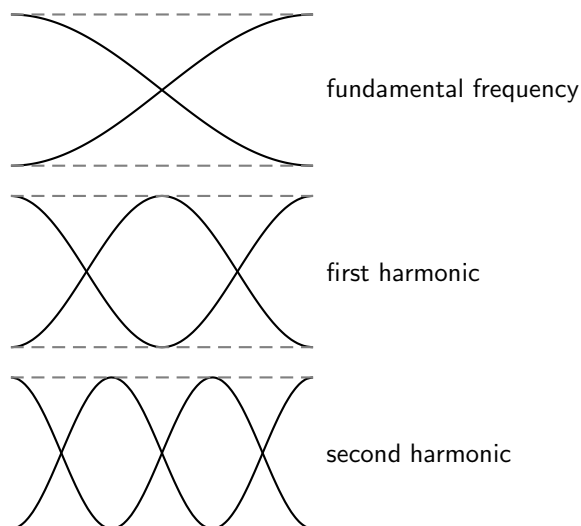


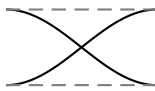
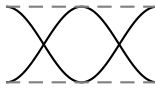
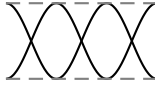
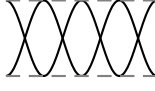
Figure 16.2: Harmonics in a pipe open at both ends.

Activity :: Investigation : Waves in a Pipe Open at Both Ends

This chart shows some standing waves in a pipe open at both ends. The pipe (shown with dashed lines) has length L .

1. Fill in the:
 - number of nodes
 - number of anti-nodes
 - wavelength in terms of L

The first and last waves are done for you.

Wave	Nodes	Antinodes	Wavelength
	1	2	$2L$
			
			
	4	5	$\frac{L}{2}$

2. Use the chart to find a formula for the wavelength in terms of the number of nodes.

The formula is different because there are more anti-nodes than nodes. The right formula is:

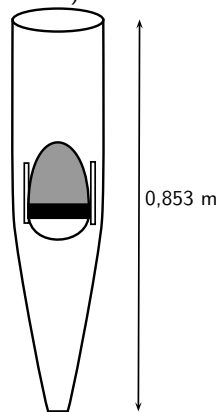
$$\lambda_n = \frac{2L}{n}$$

Here, n is still the number of nodes.



Worked Example 106: The Organ Pipe

Question: An open organ pipe is 0,853 m long. The speed of sound in air is $345 \text{ m}\cdot\text{s}^{-1}$. Can this pipe play middle C? (Middle C has a frequency of about 262 Hz)



Answer

The main frequency of a note is the fundamental frequency. The fundamental frequency of the open pipe has one node.

Step 1 : To find the frequency we will use the equation:

$$f = \frac{v}{\lambda}$$

We need to find the wavelength first.

$$\begin{aligned} \lambda &= \frac{2L}{n} \\ &= \frac{2(0,853)}{1} \\ &= 1,706 \text{ m} \end{aligned}$$

Step 2 : Now we can calculate the frequency:

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{345}{1,706} \\ &= 202 \text{ Hz} \end{aligned}$$

This is lower than 262 Hz, so this pipe will not play middle C. We will need a shorter pipe for a higher pitch.



Worked Example 107: The Flute

A flute can be modeled as a metal pipe open at both ends. (One end looks closed but the flute has an *embouchure*, or hole for the player to blow across. This hole is large enough for air to escape on that side as well.) If the fundamental note of a flute is middle C, how long is the flute? The speed of sound in air is $345 \text{ m}\cdot\text{s}^{-1}$.



Answer

We can calculate the length of the flute from $\lambda = \frac{2L}{n}$ but

Step 1 : We need to calculate the wavelength first:

$$\begin{aligned} f &= \frac{v}{\lambda} \\ 262 &= \frac{345}{\lambda} \\ \lambda &= \frac{345}{262} = 1,32 \text{ m} \end{aligned}$$

Step 2 : Using the wavelength, we can now solve for L :

$$\begin{aligned} \lambda &= \frac{2L}{n} \\ &= \frac{2L}{1} \\ L &= \frac{1,32}{2} = 0,66 \text{ m} \end{aligned}$$

Now let's look at a pipe that is open on one end and closed on the other. This pipe has a node at one end and an antinode at the other. An example of a musical instrument that has a node

at one end and an antinode at the other is a clarinet. In the activity you will find out how the wavelengths are affected.

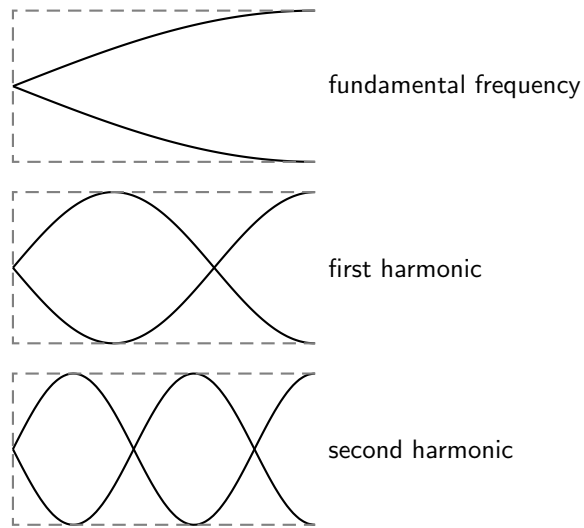


Figure 16.3: Harmonics in a pipe open at one end.


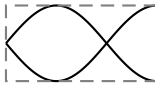
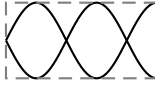

Activity :: Investigation : Waves in a Pipe open at One End

This chart shows some standing waves in a pipe open at *one* end. The pipe (shown as dashed lines) has length L .

1. Fill in the:

- number of nodes
- number of anti-nodes
- wavelength in terms of L

The first and last waves are done for you.

Wave	Nodes	Antinodes	Wavelength
	1	1	$4L$
			
			
	4	4	$\frac{4L}{7}$

2. Use the chart to find a formula for the wavelength in terms of the number of nodes.

The right formula for this pipe is:

$$\lambda_n = \frac{4L}{2n - 1}$$

380

A long wavelength has a low frequency and low pitch. If you took your pipe from the last example and covered one end, you should hear a much lower note! Also, the wavelengths of the harmonics for this tube are *not* integer multiples of each other.



Worked Example 108: The Clarinet

Question: A clarinet can be modeled as a wooden pipe closed on one end and open on the other. The player blows into a small slit on one end. A reed then vibrates in the mouthpiece. This makes the standing wave in the air. What is the fundamental frequency of a clarinet 60 cm long? The speed of sound in air is $345 \text{ m}\cdot\text{s}^{-1}$.

Answer

Step 1 : Identify what is given and what is asked:

We are given:

$$\begin{aligned} L &= 60 \text{ cm} \\ v &= 345 \text{ m}\cdot\text{s}^{-1} \\ f &= ? \end{aligned}$$

Step 2 : To find the frequency we will use the equation $f = \frac{v}{\lambda}$ but we need to find the wavelength first:

$$\begin{aligned} \lambda &= \frac{4L}{2n-1} \\ &= \frac{4(0,60)}{2(1)-1} \\ &= 2,4 \text{ m} \end{aligned}$$

Step 3 : Now, using the wavelength you have calculated, find the frequency:

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{345}{2,4} \\ &= 144 \text{ Hz} \end{aligned}$$

This is closest to the D below middle C. This note is one of the lowest notes on a clarinet.



Extension: Musical Scale

The 12 tone scale popular in Western music took centuries to develop. This scale is also called the 12-note Equal Tempered scale. It has an octave divided into 12 steps. (An **octave** is the main interval of most scales. If you double a frequency, you have raised the note one octave.) All steps have equal ratios of frequencies. But this scale is not perfect. If the octaves are in tune, all the other intervals are slightly mistuned. No interval is badly out of tune. But none is perfect.

For example, suppose the base note of a scale is a frequency of 110 Hz (a low A). The first harmonic is 220 Hz. This note is also an A, but is one octave higher. The second harmonic is at 330 Hz (close to an E). The third is 440 Hz (also an A). But not all the notes have such simple ratios. Middle C has a frequency of about 262 Hz. This is not a simple multiple of 110 Hz. So the interval between C and A is a little out of tune.

Many other types of tuning exist. Just Tempered scales are tuned so that all intervals are simple ratios of frequencies. There are also equal tempered scales with more or less notes per octave. Some scales use as many as 31 or 53 notes.

16.4 Resonance

Resonance is the tendency of a system to vibrate at a maximum amplitude at the natural frequency of the system.

Resonance takes place when a system is made to vibrate at its natural frequency as a result of vibrations that are received from another source of the same frequency. In the following investigation you will measure the speed of sound using resonance.

Activity :: Experiment : Using resonance to measure the speed of sound

Aim:

To measure the speed of sound using resonance

Apparatus:

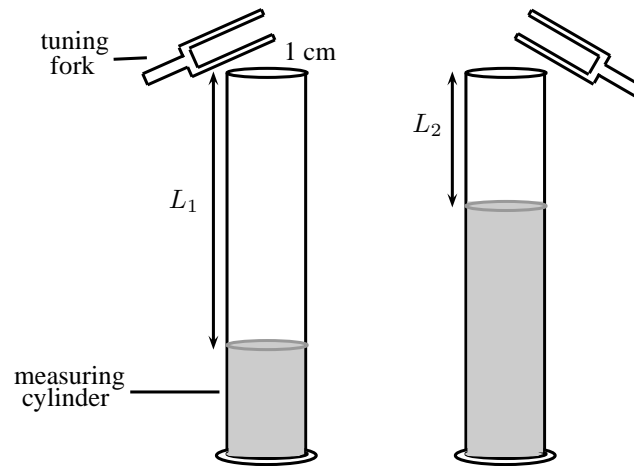
- one measuring cylinder
- a high frequency (512 Hz) tuning fork
- some water
- a ruler or tape measure

Method:

1. Make the tuning fork vibrate by hitting it on the sole of your shoe or something else that has a rubbery texture. A hard surface is not ideal as you can more easily damage the tuning fork.
2. Hold the vibrating tuning fork about 1 cm above the cylinder mouth and start adding water to the cylinder at the same time. Keep doing this until the first resonance occurs. Pour out or add a little water until you find the level at which the loudest sound (i.e. the resonance) is made.
3. When the water is at the resonance level, use a ruler or tape measure to measure the distance (L_A) between the top of the cylinder and the water level.
4. Repeat the steps ?? above, this time adding more water until you find the next resonance. Remember to hold the tuning fork at the same height of about 1 cm above the cylinder mouth and adjust the water level to get the loudest sound.
5. Use a ruler or tape measure to find the new distance (L_B) from the top of the cylinder to the new water level.

Conclusions:

The difference between the two resonance water levels (i.e. $L = L_A - L_B$) is half a wavelength, or the same as the distance between a compression and rarefaction. Therefore, since you know the wavelength, and you know the frequency of the tuning fork, it is easy to calculate the speed of sound!



Interesting fact: Soldiers march out of time on bridges to avoid stimulating the bridge to vibrate at its natural frequency.



Worked Example 109: Resonance

Question: A 512 Hz tuning fork can produce a resonance in a cavity where the air column is 18,2 cm long. It can also produce a second resonance when the length of the air column is 50,1 cm. What is the speed of sound in the cavity?

Answer

Step 1 : Identify what is given and what is asked:

$$\begin{aligned} L_1 &= 18,2 \text{ cm} \\ L_2 &= 50,3 \text{ cm} \\ f &= 512 \text{ Hz} \\ v &= ? \end{aligned}$$

Remember that:

$$v = f \times \lambda$$

We have values for f and so to calculate v , we need to first find λ . You know that the difference in the length of the air column between two resonances is half a wavelength.

Step 2 : Calculate the difference in the length of the air column between the two resonances:

$$L_2 - L_1 = 32,1 \text{ cm}$$

Therefore $32,1 \text{ cm} = \frac{1}{2} \times \lambda$

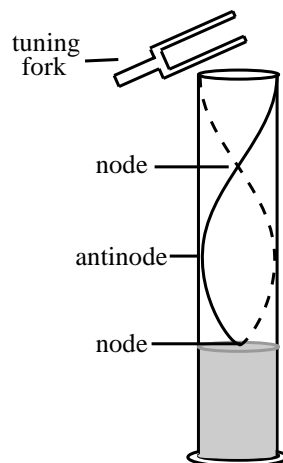
So,

$$\begin{aligned} \lambda &= 2 \times 32,1 \text{ cm} \\ &= 64,2 \text{ cm} \\ &= 0,642 \text{ m} \end{aligned}$$

Step 3 : Now you can substitute into the equation for v to find the speed of sound:

$$\begin{aligned}v &= f \times \lambda \\ &= 512 \times 0,642 \\ &= 328,7 \text{ m.s}^{-1}\end{aligned}$$

From the investigation you will notice that the column of air will make a sound at a certain length. This is where resonance takes place.



16.5 Music and Sound Quality

In the sound chapter, we referred to the quality of sound as its tone. What makes the tone of a note played on an instrument? When you pluck a string or vibrate air in a tube, you hear mostly the fundamental frequency. Higher harmonics are present, but are fainter. These are called **overtone**s. The tone of a note depends on its mixture of overtones. Different instruments have different mixtures of overtones. This is why the same note sounds different on a flute and a piano.

Let us see how overtones can change the shape of a wave:

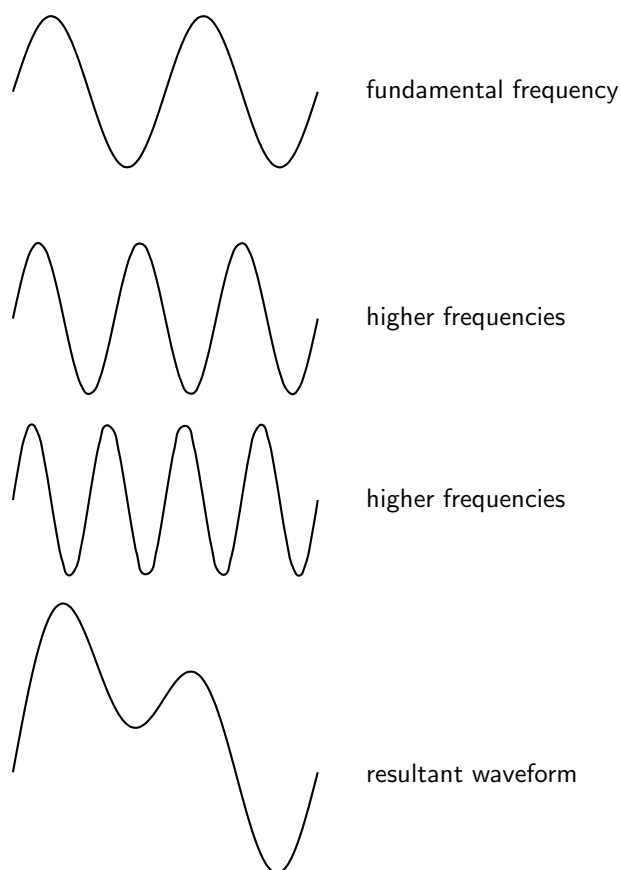


Figure 16.4: The quality of a tone depends on its mixture of harmonics.

The resultant waveform is very different from the fundamental frequency. Even though the two waves have the same main frequency, they do not sound the same!

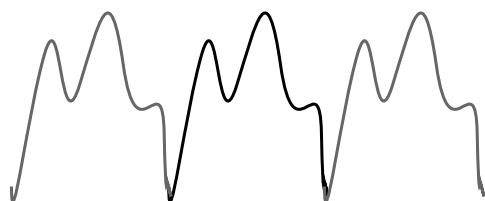
16.6 Summary - The Physics of Music

1. Instruments produce sounds because they form standing waves in strings or pipes.
2. The fundamental frequency of a string or a pipe is its natural frequency. The wavelength of the fundamental frequency is twice the length of the string or pipe.
3. The first harmonic is formed when the standing wave forms one whole wavelength in the string or pipe. The second harmonic is formed when the standing wave forms $1\frac{1}{2}$ wavelengths in the string or pipe.
4. The frequency of a standing wave can be calculated with the equation $f = \frac{v}{\lambda}$.
5. The wavelength of a standing wave in a string fixed at both ends can be calculated using $\lambda_n = \frac{2L}{n-1}$.
6. The wavelength of a standing wave in a pipe with both ends open can be calculated using $\lambda_n = \frac{2L}{n}$.
7. The wavelength of a standing wave in a pipe with one end open can be calculated using $\lambda_n = \frac{4L}{2n-1}$.
8. Resonance takes place when a system is made to vibrate at its own natural frequency as a result of vibrations received from another source of the same frequency.



Extension: Waveforms

Below are some examples of the waveforms produced by a flute, clarinet and saxophone for different frequencies (i.e. notes):



Flute waveform
B₄, 247 Hz



Clarinet waveform
E_b, 156 Hz



Saxophone waveform
C₄, 256 Hz

16.7 End of Chapter Exercises

1. A guitar string with a length of 70 cm is plucked. The speed of a wave in the string is $400 \text{ m}\cdot\text{s}^{-1}$. Calculate the frequency of the first, second, and third harmonics.
2. A pitch of Middle D (first harmonic = 294 Hz) is sounded out by a vibrating guitar string. The length of the string is 80 cm. Calculate the speed of the standing wave in the guitar string.
3. A frequency of the first harmonic is 587 Hz (pitch of D5) is sounded out by a vibrating guitar string. The speed of the wave is $600 \text{ m}\cdot\text{s}^{-1}$. Find the length of the string.
4. Two notes which have a frequency ratio of 2:1 are said to be separated by an octave. A note which is separated by an octave from middle C (256 Hz) is
 - A 254 Hz
 - B 128 Hz
 - C 258 Hz
 - D 512 Hz
5. Playing a middle C on a piano keyboard generates a sound at a frequency of 256 Hz. If the speed of sound in air is $345 \text{ m}\cdot\text{s}^{-1}$, calculate the wavelength of the sound corresponding to the note of middle C.
6. What is resonance? Explain how you would demonstrate what resonance is if you have a measuring cylinder, tuning fork and water available.
7. A tuning fork with a frequency of 256 Hz produced resonance with an air column of length 25,2 cm and at 89,5 cm. Calculate the speed of sound in the air column.

Appendix A

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